**Bonus Problems**

These bonus problems are worth points toward the Daily Work portion of your grade. The point value is listed with the problems. You must show your work to receive full credit. **No problems will be accepted after 11 AM on Monday, May 8, 2006.**

1. (5 points each) Determine whether the series converges or diverges. If it is convergent, find its sum.
   
   (a) \[ \sum_{n=1}^{\infty} \frac{(-6)^{n-1}}{5^{n-1}} \]
   
   (b) \[ \sum_{n=0}^{\infty} \frac{1}{\sqrt{2^n}} \]
   
   (c) \[ \sum_{n=1}^{\infty} \frac{e^n}{3^{n-1}} \]

2. (5 points) Express the number \( 0.73 = 0.7373737373 \ldots \) as a ratio of integers.

3. (8 points) For what values of \( p \) does the series \[ \sum_{n=2}^{\infty} \frac{1}{n^p \ln n} \] converge?

4. (5 points) For which positive integers \( k \) is the series \[ \sum_{n=1}^{\infty} \frac{(n!)^2}{(kn)!} \] convergent?

5. (5 points each) Every other even \#2-28 on p. 784

6. (8 points) If \( k \) is a positive integer, find the radius of convergence of the series \[ \sum_{n=0}^{\infty} \frac{(n!)^k}{(kn)!} x^n. \]

7. A triangle is formed with vertices \( A = (1, 2, -3), B = (3, 4, -2) \) and \( C = (3, -2, 1) \).
   
   (a) (4 points) Is triangle \( ABC \) a right triangle?
   
   (b) (4 points) Is it an isosceles triangle?

8. (5 points) Suppose \( \nabla f(2, 4) \) of a function \( f(x, y) \) has length equal to 5. Is there a direction \( u \) so that \( D_u f(2, 4) = 7 \)?

9. (8 points) If \( r = <x, y, z>, a = <a_1, a_2, a_3> \) and \( b = <b_1, b_2, b_3> \), show that the vector equation \((r - a) \cdot (r - b) = 0\) represents a sphere, and find it center and radius.

10. (8 points) If \( c = |a|b + |b|a \) where \( a, b, \) and \( c \) are all nonzero vectors, show that \( c \) bisects the angle between \( a \) and \( b \).

11. (5 points) Find the value of \( C \) if the plane \( x + 5y + Cz + 6 = 0 \) is perpendicular to the plane \( 4x - y + z = 17 = 0. \)
12. (5 points each) Change the given equation into the corresponding equation in the given coordinates.

(a) $\varphi = \frac{\pi}{6}$ to rectangular coordinates
(b) $r = 2 \sin(2\theta)$ to rectangular coordinates
(c) $\rho = 3 \cos \varphi$ to rectangular coordinates
(d) $2x^2 + 2y^2 - 4z^2 = 0$ to spherical coordinates
(e) $\rho = 2 \cos \varphi$ to cylindrical coordinates
(f) $\rho \sin \varphi = 1$ to rectangular coordinates

13. (8 points) Sketch the solid consisting of all points with spherical coordinates $(\rho, \theta, \varphi)$ such that $0 \leq \theta \leq \pi/2$, $0 \leq \varphi \leq \pi/6$ and $0 \leq \rho \leq 2 \cos \varphi$.

14. (8 points) Two particles travel along the space curves

\[ \mathbf{r}_1(t) = < t, t^2, t^3 > \quad \mathbf{r}_2(t) = < 1 + 2t, 1 + 6t, 1 + 14t > \]

Do the particles collide? Do their paths intersect?

15. (10 points) Find the curvature of the curve $x = \sinh t$, $y = \cosh t$, $z = t$ at the point $(0, 1, 0)$.

16. (5 points) Use spherical coordinates to find

\[ \lim_{(x,y,z) \to (0,0,0)} \frac{xyz}{x^2 + y^2 + z^2} \]

17. (5 points) Show that $u = (x - at)^6 + (x + at)^6$ is a solution to the wave equation $u_{tt} = a^2 u_{xx}$ where $a$ is a constant.

18. (8 points) Find the directions in which the directional derivative of $f(x, y) = x^2 + \sin(xy)$ at the point $(1, 0)$ has value 1.

19. (12 points) Show that the product of the $x$-, $y$- and $z$-intercepts of any tangent plane to the surface $xyz = c^3$ is a constant.

20. (12 points) Find the points on the surface $x^2 y^2 z = 1$ that are closest to the origin.

21. (12 points) Find the volume of the largest rectangular box with edges parallel to the axes that can be inscribed in a general ellipse

\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1. \]

22. (5 points) Find the volume of the solid that lies under the hyperbolic paraboloid $z = 4 + x^2 - y^2$ and above the square $R = [-1, 1] \times [0, 2]$.

23. (12 points each) Find the volume of the given solid.

(a) Under the surface $z = 2x + y^2$ and above the region bounded by $x = y^2$ and $x = y^3$

(b) Bounded by the cylinders $x^2 + y^2 = r^2$ and $y^2 + z^2 = r^2$
(c) Inside the sphere $x^2 + y^2 + z^2 = 16$ and outside the cylinder $x^2 + y^2 = 4$
(d) Bounded by the paraboloids $z = 3x^2 + 3y^2$ and $z = 4 - x^2 - y^2$

24. (8 points) In evaluating a double integral over a region $D$, a sum of iterated integrals was obtained as follows:

$$
\iint_D f(x, y) \, dA = \int_0^1 \int_0^{2y} f(x, y) \, dx \, dy + \int_1^3 \int_0^{3-y} f(x, y) \, dx \, dy
$$

Sketch the region $D$ and express the double integral as an iterated integral with reversed order of integration.

25. (10 points) Find the surface area of the part of the surface $z = 1 + 3x + 2y^2$ that lies above the triangle with vertices $(0, 0), (0, 1)$ and $(2, 1)$.

26. (12 points) Find the surface area of the part of the sphere $x^2 + y^2 + z^2 = 4z$ that lies inside the paraboloid $z = x^2 + y^2$.

27. (12 points) Find the surface area of the part of the cone $z^2 = a^2(x^2 + y^2)$ between the planes $z = 1$ and $z = 2$.

28. (12 points) Find the volume of the solid that lies within both the cylinder $x^2 + y^2 = 1$ and the sphere $x^2 + y^2 + z^2 = 4$.

29. (10 points) Find the volume of the solid that lies within the sphere $x^2 + y^2 + z^2 = 4$, above the $xy$-plane and below the cone $z = \sqrt{x^2 + y^2}$.

30. (10 points each) Evaluate the multiple integral.

(a) $\iint_D \frac{1}{1 + x^2} \, dA$ where $D$ is the triangular region with vertices $(0, 0), (1, 1)$ and $(0, 1)$

(b) $\iint_D y \, dA$ where $D$ is the region in the first quadrant that lies above the hyperbola $xy = 1$ and the line $y = x$ and below the line $y = 2$

(c) $\iint_D x \, dA$ where $D$ is the region in the first quadrant that lies between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 2$

(d) $\iiint_T xy \, dV$ where $T$ is the solid tetrahedron with vertices $(0, 0, 0), (\frac{1}{3}, 0, 0), (0, 1, 0)$ and $(0, 0, 1)$